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Analysis of a MEMS-based ring oscillator

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Abstract— This work introduces a MEMS oscillator composed exclusively of mechanical switches as logic components. The electromechanical model of the system is developed and the conditions for a periodic response are established.

I. INTRODUCTION

Microelectromechanical systems (MEMS) have been proposed as an alternative to standard CMOS-based switches due to their ideally zero leakage current and abrupt switching behaviour [1, 2]. In [3], the basis for several MEMS-based logic components are laid and several implementations of inverters or NAND gates are proposed. The parallelism between CMOS and MEMS switches has also been extended to fields which are traditionally dominated by the CMOS technology, such as logic circuits [4] or memory cells [5]. With logic circuits in mind, one may wonder whether a purely *MEMS-based synchronous logic* can be achieved, i.e. can a digital clock signal be generated using only MEMS switches? And can this clock signal be used to drive other MEMS-based logic components? The aim of this paper is to show how such a classical concept as the CMOS ring oscillator (a loop consisting of an odd number of inverters in series) may be extended into the MEMS domain and in which respect we may answer positively to the first of these two questions. In section II, the electromechanical model of a MEMS cantilever switch is presented. In section III, we show under what hypotheses a MEMS-based ring oscillator can be modelled as an autonomous impact oscillator [6, 7] and what the conditions for the existence of a periodic regime are. In section IV, the results obtained in section III under some simplifying assumptions and those obtained from the simulation of the device are compared and some general design guidelines are discussed.

II. ELECTROMECHANICAL MODEL OF THE SWITCH

Fig. 1 represents the studied out of plane cantilever switch configuration. When a sufficient voltage is applied across gate and source, the flexible part of the source bends as a result of electrostatic forces until its tip touches the drain and an electrical contact is established. The simplest MEMS inverter consists of two switches, PMEMS and NMEMS, respectively with source voltages V_{dd} and V_{ss} , having a common drain and a common gate (Fig. 2). With correct

design [2-5], the drain (output) voltage is V_{dd} when the gate (input) voltage is V_{ss} and vice-versa. Putting an odd number of such inverters in series and closing the loop results in an intrinsically unstable system, known as a ring oscillator, a simplified model of which is established in the next sub-sections.

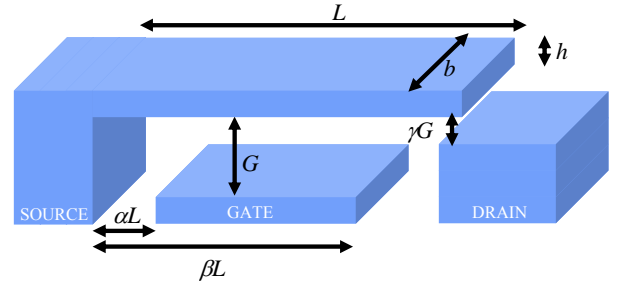


Figure 1. Schematic of a MEMS cantilever switch and notations.

A. Electrical model

In a ring oscillator, the switches are either in the “free mode”, such as NMEMS in Fig. 2 which is moving toward or away from the drain, or in the contact mode, such as PMEMS in Fig. 2. In the contact mode, a resistive path between source and drain is established. As long as mechanical contact is maintained, V_D is governed by:

$$R_{on} C_{on} \frac{dV_D}{dt} + V_D = V_S, \quad (1)$$

where R_{on} is the resistance of the path between source and drain and C_{on} is the total capacitance when contact is established. R_{on} can be assimilated to the contact resistance (R_{co} in Fig 2) between source and drain, which is typically several orders of magnitude greater than the intrinsic resistance of the drain or of the source (R in Fig.2). If $\tau_{on} = R_{on} C_{on}$ is small compared to the mechanical contact duration, V_D goes to V_S . Typical values for the contact resistance in MEMS relays (taken from [8]) are in the 1-10k Ω

range (depending on the contact force and area) while the gate-source capacitance is in the 0.1-10fF range. In the free mode, provided the change in C_{SGP} or C_{SGN} is not too large compared to C_L , the gate/drain voltages can be considered as constant. Note that the behaviour of our oscillator is considerably different from that of the one proposed in [7], where the mechanical and electrical time constants have the same order of magnitude. It is assumed throughout the rest of the paper that $V_{ss}=0$.

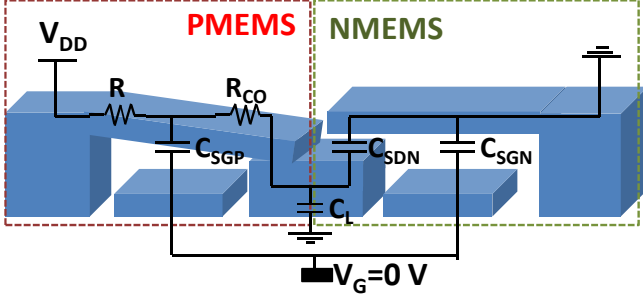


Figure 2. MEMS inverter and its electrical model when $V_G=0V$.

B. Mechanical Model

The beams in the ring oscillator can be modelled with the Euler-Bernoulli equation, in which we account for squeezed-film damping as the dominant damping phenomenon and electrostatic actuation:

$$E \frac{bh^3}{12L^4} \frac{\partial^4 w}{\partial x^4} + \mu_{eff} \frac{b^3}{G^3} \frac{\chi(\alpha, \beta)}{(1-w)^3} \frac{\partial w}{\partial t} + \rho b h \frac{\partial^2 w}{\partial t^2} = \epsilon_0 \frac{b}{2G^3} V^2 \frac{\chi(\alpha, \beta)}{(1-w)^2} + F_c(w) \delta(1) \quad (2)$$

where $x \in [0,1]$ is the normalized coordinate along the length of the beam ($x=1$ corresponding to the tip), displacement $w(x,t)$ is normalized with respect to gap G , V is the voltage applied to the beam, F_c is the contact force applied at the beam tip and $\chi(\alpha, \beta)$ is the characteristic function of the actuation electrode (equal to 1 if $x \in [\alpha, \beta]$ and 0 otherwise). E and ρ are the material's Young's modulus and mass density, μ_{eff} the effective viscosity of the surrounding medium and ϵ_0 the permittivity of vacuum. The solution of (2) can be approximated as:

$$w(x,t) = a_1(t)W_1(x) + a_2(t)W_2(x), \quad (3)$$

where

$$W_1 = \frac{1}{2}(U_1 + U_2), \quad W_2 = \frac{1}{2}(U_1 - U_2) \quad (4)$$

and $U_1(x)$ and $U_2(x)$ are the first two eigenmodes of the cantilever, with eigenvalues λ_1^4 and λ_2^4 , normalized so that $U_1(1)=U_2(1)=1$. Consequently, $W_1(1)=1$ and $W_2(1)=0$. Mechanical contact between source and drain then takes place when $a_1 \geq \gamma$, regardless of a_2 . We use the Galerkin method to derive the following nonlinear mechanical model of the beam:

$$\mathbf{K}\mathbf{a} + \mathbf{B}(\mathbf{a})\dot{\mathbf{a}} + \ddot{\mathbf{a}} = \mathbf{f}_c(\mathbf{a})V^2 + \mathbf{f}_c(\mathbf{a}), \quad (5-a)$$

where $\mathbf{a}^T = [a_1, a_2]$ and

$$\mathbf{K} = \frac{Eh^2}{192\rho L^4} \begin{bmatrix} \lambda_1^4 + \lambda_2^4 & \lambda_1^4 - \lambda_2^4 \\ \lambda_1^4 - \lambda_2^4 & \lambda_1^4 + \lambda_2^4 \end{bmatrix}, \quad (5-b)$$

$$\mathbf{B} = (B_{ij})_{1 \leq i,j \leq 2}, \quad B_{ij}(\mathbf{a}) = \frac{\mu_{eff} b^2}{\rho h G^3} \int_0^1 \frac{W_i W_j}{(1-w)^3} dx, \quad (5-c)$$

$$\mathbf{f}_c = (f_i)_{1 \leq i \leq 2}, \quad f_i = \frac{\epsilon_0}{2\rho h G^3} \int_0^1 \frac{W_i}{(1-w)^2} dx \quad (5-d)$$

$$\mathbf{f}_c = \begin{bmatrix} K_c(a_1 - \gamma) + B_c \dot{a}_1 \\ 0 \end{bmatrix} \text{ if } a_1 \geq \gamma, \quad \mathbf{0} \text{ otherwise.} \quad (5-e)$$

Note that the contact force (5-e) is not necessarily conservative. Using (1) and (5), a MEMS-based ring oscillator with an arbitrary number of stages can be simulated and several of its characteristics can be explored (influence of contact stiffness and damping, influence of supply voltage on the existence of periodic solutions, etc.). However, the model developed in this section, in spite of its apparent simplicity, is difficult to simulate because of (i) the existence of discontinuities, and of (ii) the coexistence of several phenomena with very different time scales (ideally, the transition from a high to a low state should be very short compared to the mechanical contact duration, and even shorter compared to the period of oscillation). In the following section, we propose a semi-analytical tool based on simplifying assumptions which will allow us to predict the existence of periodic regimes without resorting to computationally-intensive simulation tools.

III. SEMI-ANALYTICAL LIMIT CYCLE PREDICTION

The following set of assumptions is made. First of all, only small amplitude motion of the beam is considered, so that $\mathbf{B}(\mathbf{a}) \rightarrow \mathbf{B}(\mathbf{0})$ and $\mathbf{f}_c(\mathbf{a}) \rightarrow \mathbf{f}_c(\mathbf{0})$ in (5-a). This allows us to recast (5) in the following state-space form:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}_e V^2 + \mathbf{b}_c(\mathbf{z}), \quad (6)$$

with

$$\mathbf{z} = \begin{bmatrix} \mathbf{a} \\ \dot{\mathbf{a}} \end{bmatrix}, \mathbf{b}_e = \begin{bmatrix} \mathbf{0} \\ \mathbf{f}_e(\mathbf{0}) \end{bmatrix} \text{ and } \mathbf{A} = \begin{bmatrix} \mathbf{0}_2 & \mathbf{I}_2 \\ -\mathbf{K} & -\mathbf{B}(\mathbf{0}) \end{bmatrix}. \quad (7)$$

It is also assumed that mechanical contact is instantaneous and governed by:

$$\mathbf{z}(t_c^+) = \mathbf{S}\mathbf{z}(t_c^-), \mathbf{S} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -\kappa & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad (8)$$

where t_c designates the instant at which contact occurs. Equation (8) implies that, when a contact takes place, the state of the system is unchanged, except for the first modal velocity coefficient, governed by:

$$\dot{a}_1(t_c^+) = -\kappa \dot{a}_1(t_c^-). \quad (9)$$

Coefficient $\kappa < 1$ then represents the mechanical losses during contact. Between impacts, a beam is then governed by:

$$\dot{\mathbf{z}} = \mathbf{A}\mathbf{z} + \mathbf{b}_e V^2. \quad (10)$$

Finally, V^2 in (10) can take either of two values, ideally equal to $V_{off}^2 = 0$ and $V_{on}^2 = V_{dd}^2$, provided the electric time constant is small¹. The commutation from one value to the other takes place when the output of the previous inverter in the ring oscillator loop changes.

From this set of hypotheses, the existence of simple limit cycles can be predicted. In particular, let us assume that a limit cycle with period T , half-period $T/2$ and delay τ can take place in the system, meaning that:

$$V(t) = \begin{cases} V_{on}, t \in]0, \tau] \\ 0, t \in]\tau, \tau + T/2] \\ V_{on}, t \in]\tau + T/2, T] \end{cases} \quad (11)$$

where the origin of time is taken at the moment when the mechanical contact occurs. Note that τ depends on the number of stages in the ring oscillator: $\tau = 0$ for a one-stage oscillator, $\tau = T/3$ for a three-stage oscillator, etc. The analytical solution of (10-11) is straightforward, provided the beam does not come into contact with the drain for $t \in]0, T[$. Under this hypothesis, we find:

¹ Otherwise, the load capacitance is only partially charged or discharged. This, however, does not affect very much the rest of the analysis.

$$\mathbf{z}_f - \mathbf{e}^{\mathbf{A}T} \mathbf{z}_0 = \left(\mathbf{I}_4 - \mathbf{e}^{\mathbf{A}\left(\frac{T}{2}-\tau\right)} + \mathbf{e}^{\mathbf{A}(T-\tau)} - \mathbf{e}^{\mathbf{A}T} \right) \mathbf{z}_e V_{on}^2, \quad (12)$$

where $\mathbf{z}_f = \mathbf{z}(T^-)$, $\mathbf{z}_0 = \mathbf{z}(0^+)$, $\mathbf{z}_e = -\mathbf{A}^{-1}\mathbf{b}_e$ and $\mathbf{e}^{\mathbf{M}}$ designates the matrix exponential of \mathbf{M} . If the motion is periodic, we should have $\mathbf{z}(T^-) = \mathbf{z}(0^-)$, so that, using (8), (12) becomes:

$$\mathbf{z}_0(V_{on}^2) = (\mathbf{S}^{-1} - \mathbf{e}^{\mathbf{A}T})^{-1} \left(\mathbf{I}_4 - \mathbf{e}^{\mathbf{A}\left(\frac{T}{2}-\tau\right)} + \mathbf{e}^{\mathbf{A}(T-\tau)} - \mathbf{e}^{\mathbf{A}T} \right) \mathbf{z}_e V_{on}^2. \quad (13)$$

For a given value of γ , V_{on}^2 should then be adjusted so that:

$$[1 \ 0 \ 0 \ 0] \mathbf{z}_0 = \gamma. \quad (14)$$

Since γ and V_{on}^2 are positive, this is only possible if, for instance,

$$[1 \ 0 \ 0 \ 0] \mathbf{z}_0(1) > 0. \quad (15)$$

Furthermore, at time 0^+ , the beam should be bouncing away from the drain (and not “into it”), i.e.:

$$[0 \ 0 \ 1 \ 0] \mathbf{z}_0(1) < 0. \quad (16)$$

Finally, (13) is valid only if there are no spurious contacts for $t \in]0, T[$. There is no analytical way of verifying this, although the analytical expression of $\mathbf{z}(t)$ exists. One should then tabulate $\mathbf{z}(t_i)$ for $t_i \in]0, T[$ and verify that, $\forall i$, the first component of $\mathbf{z}(t_i)$ (corresponding to $a_1(t_i)$) is smaller than γ . If this condition is met, and (15) and (16) as well, then a limit cycle with period T , half-period $T/2$, delay τ and no supplementary impacts between 0 and T may take place in the system. By setting γ and sweeping a range of values of T , it is then possible to plot curves giving T versus V_{on} . Note that the procedure described above can be extended to the case when several impacts take place during a period. However, this requires solving a nonlinear set of equations and quickly becomes impractical when the number of impacts is more than a few.

IV. SIMULATION RESULTS

We have represented in Fig. 3 the starting transient of $a_1(t)$, obtained by simulating (1) and (5) with Matlab/Simulink. The device is a 1-stage MEMS ring oscillator consisting of two identical beams arranged as in Fig. 2, with shorted gate and drain, with the following geometry:

$\alpha=0.1$, $\beta=0.9$, $\gamma=0.15$, $L=25\mu\text{m}$, $b=1\mu\text{m}$, $h=0.25\mu\text{m}$ and $G=0.3\mu\text{m}$ ($1/f_0=1.88\mu\text{s}$, $Q_{\text{squeezed}}=15.2$, $V_{\text{snap}}=2.07\text{V}$). The material is assumed to be polysilicon and the surrounding medium air at ambient pressure. The actuation voltage is $V_{dd}=1.4\text{V}$ and we set $R_{on}=10\text{k}\Omega$ and $C_{on}=10\text{fF}$. In these conditions, the oscillator reaches a periodic regime after a short time, with a period of $T=1.15\times 10^{-6}\text{s}$. We also show in Fig.4 the steady-state behaviour of $a_1(t)$ as predicted by the method presented in section III. There is an excellent match between the simulated response and the predicted one.

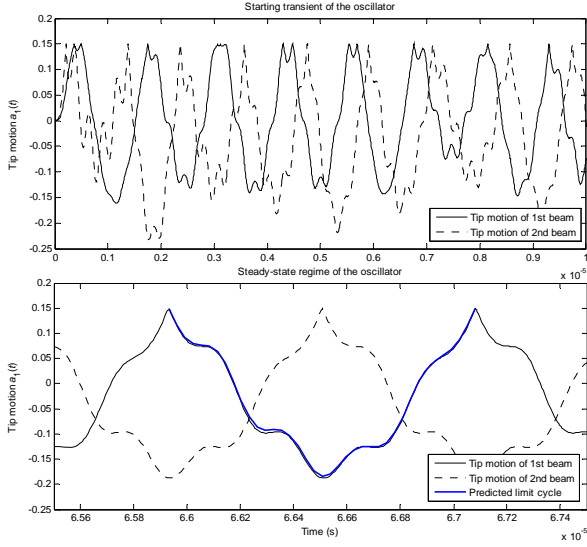


Figure 3. Simulated transient response of $a_1(t)$ (normalized beam tip position) for the two beams composing the switch (top) and simulated and predicted steady-state response (bottom).

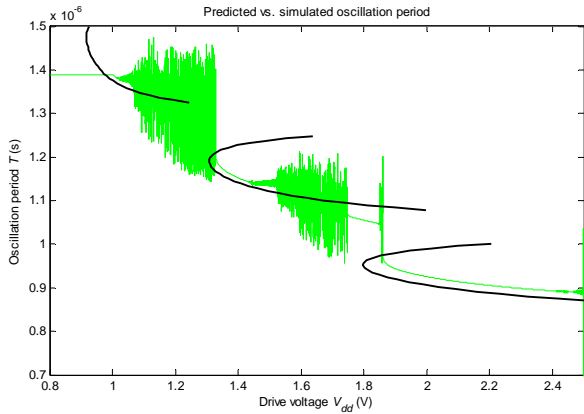


Figure 4. Comparison of predicted “ T versus V_{on} ” curve (black line) and results obtained by transient simulation (green line), starting from $V_{dd}=2.5\text{V}$.

In Fig.4, we compare the results obtained by slowly sweeping the actuation voltage in the transient nonlinear model to the “ T versus V_{on} ” curve obtained with our semi-analytical method - for the transient simulations, T is in fact the “apparent” period, i.e. the time between two successive rising edges of $V(t)$. As expected [6, 7], depending on the

value of V_{on} , the simulated behaviour is not always periodic, even though our semi-analytical method predicts one or more possible periodic limit cycles. For example, transient simulation shows that the system exhibits chaotic behaviour when V_{on} is between 1.5V and 1.75V , whereas our semi-analytical method predicts the existence of one or two periodic solutions. This illustrates the fact that the stability of the predicted limit cycles is not guaranteed *a priori* and should be the object of careful study. This situation is even more pronounced for structures with small damping factors (larger Q).

V. CONCLUSION

We have shown that under certain conditions, one may contrive to generate a periodic square signal using an odd number of MEMS inverters in a ring oscillator configuration. The governing equations of such a system were established and a semi-analytical method for predicting the existence of simple limit cycles was proposed. Comparison of the results obtained by our method and by simulation incite us to develop tools to study the stability of these limit cycles, for example following [9]. Qualitatively, we have found that too large a quality factor is detrimental to the stability of the system. Also, a possible reason for non-periodic behaviour is that the two natural resonance frequencies of the system (those of the first two eigenmodes of the beam) are not harmonically related. The beam profile could then be carefully designed, for example to tune the beam for periodic behaviour in a given voltage range. Several questions remain open, e.g. heating, friction and wear, etc. which are the subject of ongoing work.

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